The Pareto distribution for low grazing angle and high resolution X-band sea clutter

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Abstract— A radar pulse that impinges upon a radar resolution area of the sea surface produces backscattered returns which are called sea clutter. For low grazing angles and very fine radar cell resolution areas, the clutter intensity distribution departs significantly from the exponential distribution. In this case, the clutter is said to display spiky behavior and the distribution of the intensity develops a much longer tail relative to the exponential distribution. Statistical analysis of collected data near a grazing angle of $0.2\,^\circ$ at X-band from the sea off the coast of Kauai, Hawaii are examined relative to the log-normal, Weibull and K distributions. Based on an analogy of sea clutter and other disciplines including computer networks and finance, we also apply the Pareto distribution to the collected data. We also compare the data to the WW and KK two-population mixture distributions. Maximum likelihood estimation of the parameters of the distributions are obtained from the measured data. In all cases, the two population mixture distributions and the Pareto distribution are more accurate than the three classical distributions. However the Pareto distribution has the advantage of being an analytically tractable two parameter distribution while having similar accuracy to the five parameter WW and KK at critical values.

I. Introduction

The performance of high resolution radars at low grazing angles has been a topic of research for the last several decades. Extensive surveys of many of the experimental and theoretical developments are given in[1][2]. Despite the extensive number of studies, the current phenomenological understanding of sea clutter modeling is incomplete. Partially due to this incomplete knowledge, a variety of statistical models have been proposed for the distribution of the sea clutter. An accurate sea clutter model allows for the optimization of the radar detector chain including its CFAR processor. This in turn allows for enhanced target detection and better control of the false alarm rate. In this paper we assess the accuracy of some new and unexamined mixture distributions for the low grazing angle sea clutter data of [4]. Maximum likelihood estimation (MLE) of the parameters of the distributions are obtained and the results are compared to the actual data's distribution. In Section II of this paper we briefly review the measurement parameters, the collected data and sea conditions of the experiment. In Section III, we overview the traditional models as well as the mixture models[5] and Pareto distribution[7]. In Section IV we estimate the parameters of the distributions using the MLE and compare their respective CDFs. Conclusion of this paper is given in Section V.

II. Experimental Measurements

An itemized description of the measurement radar appears in Table I. The Data was collected in Kauai, Hawaii between August and December of 1994 by the Naval Research Laboratory (NRL), in conjunction with Naval Air Warfare Center, China Lake (NAWC/CL), the Johns Hopkins University Applied Physics Laboratory (JHU/APL), and Texas Instruments (TI), since then acquired by Raytheon. There are several descriptors to note about the data. The grazing angle, which is a function of the radar height and radar geometry is around 0.22 degrees. The data was collected for both VV and HH. However the Radar was not dual-polarized. Consequently, the HH and VV data collections were measured separately. The Radar Height was 23 meters and the radar range was 5.74 km for the vertical/vertical (VV) measurements and 6.11 km for the horizontal/horizontal (HH) measurements. The second major descriptor of the measured data is the transmit geometry. Both upwind (UP) and crosswind (CR) measurements were obtained. However, in this paper, we have only concentrated on the upwind (UP) measurements as they exhibit more of the "spiky" characteristics which makes sea clutter difficult to model at low grazing angles. During the course of the clutter measurements, the wind speed measured was about 9 m/s, while the largest significant wave height (average height of the $1/3$ highest waves) measured was around 3 m. These conditions indicate a hydrographic sea state of 4. In Figure 1, we have presented the calibrated RCS in decibels above a square meter (dBsm) as a function of time in seconds along the horizontal axis and of range, in meters, along the vertical axis. These are also known as range-time intensity (RTI) plots. In Figure 2, we have plotted the variation of the HH and VV RCS for a specific range cell over a 5 seconds. As noted in [4], the HH clutter is sharper, spikier and more intermittent. In Figure 3, the temporal variation of the spectral frequency content is displayed for the range cell of Figure 2 in the following manner. The individual spectra are generated by diving the entire time interval into consecutive, non-overlapping windows, and then calculating the power spectral density (PSD) within each Hanning-weighted window. The set of power spectra is then normalized by dividing each value in every PSD by the largest value that occurred in entire set; the new maximum is 1. Finally, the entire sequence of normalized power spectral spanning the entire time interval is displayed in a 3-D plot.

Report Documentation Page		*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE **MAY 2010**	2. REPORT TYPE	3. DATES COVERED **00-00-2010 to 00-00-2010**

4. TITLE AND SUBTITLE **The Pareto distribution for low grazing angle and high resolution X-band sea clutter**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Naval Research Laboratory,Washington,DC,20375**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES
See also ADM002322. Presented at the 2010 IEEE International Radar Conference (9th) Held in Arlington, Virginia on 10-14 May 2010. Sponsored in part by the Navy.

14. ABSTRACT
A radar pulse that impinges upon a radar resolution area of the sea surface produces backscattered returns which are called sea clutter. For low grazing angles and very fine radar cell resolution areas, the clutter intensity distribution departs significantly from the exponential distribution. In this case, the clutter is said to display spiky behavior and the distribution of the intensity develops a much longer tail relative to the exponential distribution. Statistical analysis of collected data near a grazing angle of 0.2 ◦ at X-band from the sea off the coast of Kauai Hawaii are examined relative to the log-normal, Weibull and K distributions. Based on an analogy of sea clutter and other disciplines including computer networks and finance, we also apply the Pareto distribution to the collected data. We also compare the data to the WW and KK two-population mixture distributions. Maximum likelihood estimation of the parameters of the distributions are obtained from the measured data. In all cases, the two population mixture distributions and the Pareto distribution are more accurate than the three classical distributions. However the Pareto distribution has the advantage of being an analytically tractable two parameter distribution while having similar accuracy to the five parameter WW and KK at critical values.

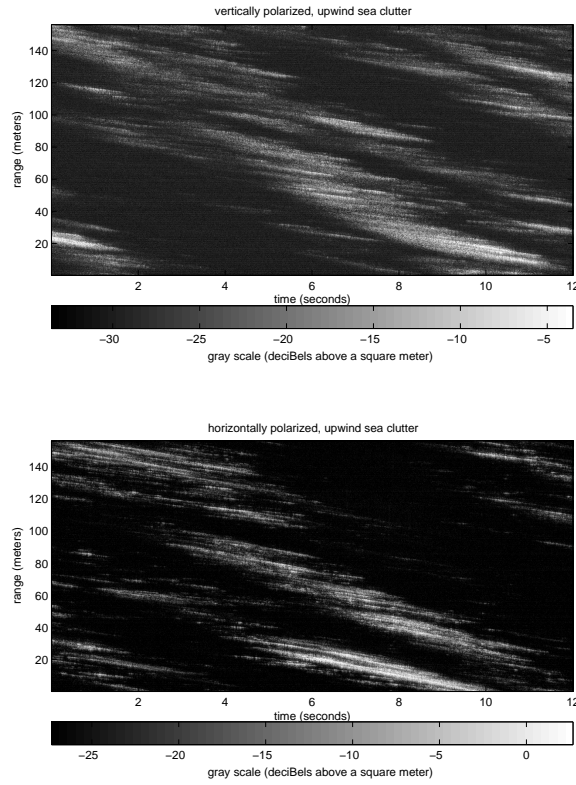| 15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**5**	

Fig. 1.   RTI plots of radar cross section in dB above a square meter for VV and HH
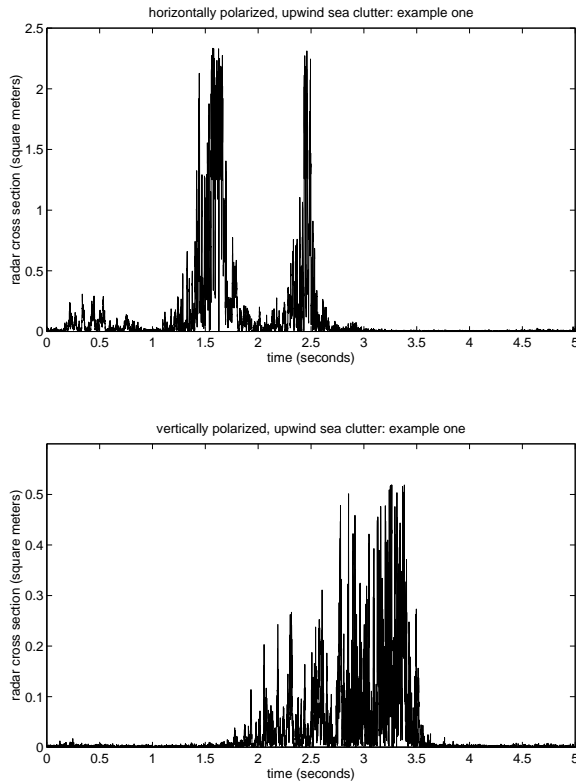

Fig. 2.   Radar cross section in square meters for a specific range of the HH and VV data
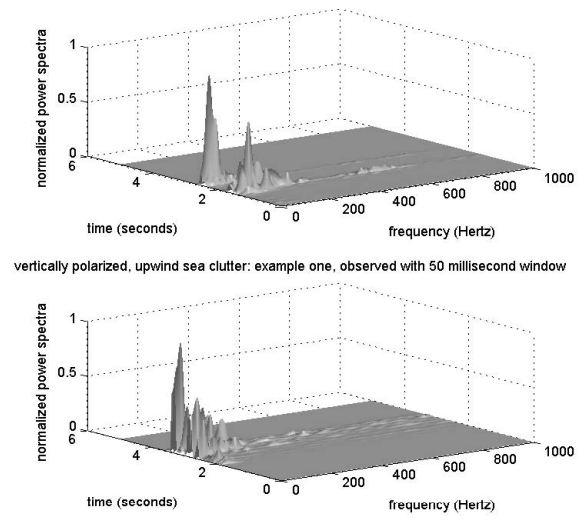

Fig. 3.   Normalized power spectra for HH and VV for a specific range cell

TABLE I

RADAR DESCRIPTORS

Operation Frequency	X-band (9.5-10.0 Ghz)
Average Power	500 W
Pulse Length	2.5 ns compressed
Pulse Repitition Frequency (PRF)	2000 Hz
Data Collection Extent	156 m range swath
Data Collection Mode	Fixed antenna pointing
Azimuth Beamwidth	2.4 °
Elevation Beamwidth	4.0 °
Polarization	VV and HH

III. DISTRIBUTION MODELS

A variety of distribution models have been proposed to model distribution of the sea clutter amplitude or intensity returns. Among these, the K[1], Log-Normal[3] and Weibull[2] distributions have been cited frequently and partial success with these models have been reported. From these, the K distribution which is a compound-Gaussian model[13] [14] has been given a phenomenological interpretation. According to this interpretation, the K distribution envelope is a compound distribution consisting of a locally Rayleigh distribution speckle whose mean is modulated by a gamma distribution. The speckle part is assumed to be due to the scattering primarily by the capillary waves and ripples while the underlying mean (also called the texture) of the patch is assumed to be from the ocean gravity waves. Instead of the gamma distribution, other distributions for the texture such as lognormal, generalized Gaussian and inverse gamma have been proposed [13] [14]. In their work [13], it was noted that for a range resolution of 3 meters, the compound-Gaussian and the K distribution did not provide a good fit. The K distribution has also been extended to take into account different types of "spiky" phenomenon[1] such as discrete spikes (bursts and whitecaps). For example, the compound K-A distribution

consists of the Poisson distribution, gamma distribution and the Rayleigh distributions[1]. Taking a different approach, Dong has recently [5] proposed the KK and Weibull-Weibull (WW) distributions for high resolution and high grazing angle sea clutter. His results indicates that the KK, WW and KA perform similarly in such a scenario. However to estimate the parameters of the KK and WW is simpler than the K-A distribution. These mixtures distribution can be given the interpretation of a two-population model. That is with a probability of p the clutter belongs to a certain distribution with a CDF $F_1(z)$ and with a probability of $1-p$, the clutter belongs to another distribution with a CDF $F_2(z)$. Here, we briefly overview the distributions used in this paper.

The Probability Density Function (PDF) of the log-normal distribution is given by[3]

$$f_L(z) = \frac{1}{z\sigma\sqrt{2\pi}} \exp -\frac{(ln(z)-\mu)^2}{2\sigma^2} \qquad (1)$$

where z is lognormal if and only if $ln(z)$ is a Gaussian with a mean μ, and variance σ. The Cumulative Distribution Function (CDF) $F_L(a)$ is of the form:

$$F_L(a) = \Phi(\frac{ln(a)-\mu}{\sigma}) \qquad (2)$$

where Φ is the standard normal function.

The PDF of the K distribution is given by[1]:

$$f_K(z) = \frac{2b^{\frac{v+1}{2}} z^{\frac{v-1}{2}}}{\gamma(\nu)} K_{v-1}(2\sqrt{zb}) \qquad (3)$$

where z denotes the intensity of the sea clutter, ν is the shape parameter, b is the scale parameter. The CDF $F_K(a)$ is of the form:

$$F_K(z) = 1 - \frac{2b^{\frac{v}{2}} z^{\frac{v}{2}}}{\gamma(\nu)} K_v(2\sqrt{bz}) \qquad (4)$$

Details about the K distribution can be found in [1].

The Weibull distribution which contains the Rayleigh distribution as a special case is another popular distribution for sea clutter[3]. The PDF of the Weibull distribution is given by[1]:

$$f_W(z) = \frac{\gamma}{\varpi}(\frac{a}{\varpi})^{\gamma-1} \exp(-(\frac{a}{\varpi})^\gamma) \qquad (5)$$

where z denotes the intensity of the sea clutter, γ is the shape parameter, ϖ is the scale parameter. The CDF $F_W(z)$ of the Weibull is:

$$F_W(z) = 1 - \exp(-(\frac{a}{\varpi})^\gamma) \qquad (6)$$

The WW distribution has the PDF:

$$f_{WW}(z) = pf_{W1}(z) + (1-p)f_{W2}(z) \qquad (7)$$

and its CDF:

$$F_{WW}(z) = pF_{W1}(z) + (1-p)F_{W2}(z) \qquad (8)$$

where the subscripts indicate two Weibull distributions that may have different shape and scale parameters.

The KK distribution is defined similarly, except the mixtures are K distributions instead of Weibull distributions.

The PDF of the Generalized Pareto distribution is defined as[7]:

$$f_{GP}(z) = \frac{1}{\lambda}(1 - k\frac{z}{\lambda})^{(\frac{1}{k}-1)} \qquad (9)$$

where k is the shape parameter and λ is the scale parameter. The value for z is $0 \le z < \infty$ for $k \le 0$ and $0 \le z < \frac{\lambda}{k}$ for $k > 0$. When $k = 0$, the Generalized Pareto distribution takes the form of an exponential PDF with the mean λ. When $k < 0$, the generalized Pareto distribution takes the form of Pareto distribution. When $k = 1$, it takes the form of a uniform random variable on $[0, \lambda]$. For this paper, k is always less than zero and z denotes the intensity. The Pareto distribution is also a continuous mixture of exponential distributions with gamma mixture parameters and it is formulated as:

$$f_P(z;k) = \int_0^\infty f_E(z;\xi)f_G(\xi;k,\lambda)\,d\xi \qquad (10)$$

where f_E is an exponential PDF with a mean $\frac{1}{\xi}$ and f_G is a gamma pdf with parameters k, λ. From a derivation point of view, the difference between the Pareto distribution and the K distribution is that the reciprocal of the mean of the exponential in the Pareto distribution is conditioned by the gamma distribution. In the K distribution, it is the mean of the Rayleigh (for envelope) and exponential (for intensity) that is conditioned by the gamma [1]. The Pareto distribution has some shared properties including the power-law property of the Tsallis distribution which was proposed in [12] for modeling the differentiated data of sea clutter.

It should be noted that the Pareto distribution has only two parameters compared to the five parameters of the WW and KK distributions. The Pareto distribution has been used for heavy tailed problems in different fields including physics, economics and finance[11]. In computer networks, it has been shown that data traffic may exhibit bursts of high data rates even when the average data rate is low[10]. The Pareto distribution is more accurate than the traditional Poisson and Binomial distributions for such a case[10]. This is analogous to sea clutter where the average clutter return could be low but bursts of spikes are exhibited. An interpretation is that the Pareto distribution follows the power law[11], where the probability of measuring a value varies inversely as a power (with an exponent factor) of that value.

IV. ANALYSIS OF THE DATA

To obtain the parameters for a distribution f_Z from the data, we have used maximum likelihood estimator (MLE)[8]. In [14], it was shown that for realistic sea clutter, the MLE has a smaller mean square error (MSE) than the method of moments (MoM) and method of fractional moments (MoFM) estimators.

For the case of independent and identically distributed samples, if N independent samples $z_1, z_2..., z_n$ are drawn from

a distribution with parameters $\boldsymbol{\theta}$, then their likelihood function is defined as:

$$L(\boldsymbol{\theta}) = \prod_{n=1}^{N} f_Z(z_i; \boldsymbol{\theta_{ML}}) \qquad (11)$$

The maximum likelihood estimate of $\boldsymbol{\theta}$ denoted by $\boldsymbol{\theta_{ML}}$ is given by:

$$\boldsymbol{\theta_{ML}} = \arg\max_{\boldsymbol{\theta}} L(\boldsymbol{\theta}) \qquad (12)$$

In order to ensure that the joint PDF can be factored into independent and identically distributed PDFs, the data was sub-sampled uniformly by a factor much greater than its significant correlation length. However, to ensure accuracy, the number of data points used for the estimation was near a half million samples. For the Log-Normal distribution, the MLE has a closed form solution. For the K, WW and KK, we have used the constrained optimization method noted in [6]. In all cases, the log-likelihood[8] which gives numerically more stable results were used. For the Generalized Pareto method, the MLE may be found by a constrained optimization problem using the Nelder-Mead algorithm[9].

Before calculating the MLE of the parameters, we normalized the clutter intensity by one. This does not change the shape parameter of the estimators but was mainly done so that the slope of the HH and VV CDFs can be compared on the same axis. For the actual RCS values of the data, the reader is referred to [4]. Figures 4 and 5 shows a plot of the CDFs with the MLE parameters for the HH data. Figures 6 and 7 shows the CDFs with the MLE for the VV data.

In Tables 2 and 3 we have calculated the difference in dB between the true and estimated intensity for some intermediate and critical CDF values. For the case of two parameter distributions, the Pareto has the best fit for the HH and VV data. It also has the smallest maximum deviation among the two parameter distributions. Compared to the WW and KK distributions, at a CDF value of 0.99, the Pareto is about 3 dB worse for HH and 1.5 dB worse for VV. However at CDF value of 0.999, it performs better than the WW and has similar performance to the KK. We should note that WW and KK are five parameters distributions whereas the Pareto is a two parameter distribution. Consequently, one would expect that a distribution with five parameters outperforms a distribution with two parameters. However, the Pareto distribution outperformed all the two parameter distributions and its performance is virtually the same as the WW and KK at the critical point of 0.999.

V. CONCLUSION

Several distributions were examined for high resolution and low grazing angle sea clutter data. MLE estimation was performed to obtain the parameters of all the distributions. Despite the fact that the Pareto distribution is a two parameter distribution, it outperformed the W, K, log-normal and WW distributions in the tail region and its performance was very close to the KK. We also note that the Pareto distribution

TABLE II
CDF ESTIMATION ERRORS FOR HH IN dB

CDF value	0.25	0.5	0.75	0.90	0.99	0.999
Weibull	2.02	1.08	2.88	2.23	4.72	8.72
Lognormal	0.20	0.61	1.17	0.30	5.63	8.12
K	0.21	1.54	2.55	1.5	5.74	9.79
WW	0.39	0.30	0.14	0.21	0.52	3.31
KK	0.21	0.21	0.19	0.28	0.37	2.81
P	0.65	0.21	1.00	0.53	3.39	2.74

TABLE III
CDF ESTIMATION ERRORS FOR VV IN dB

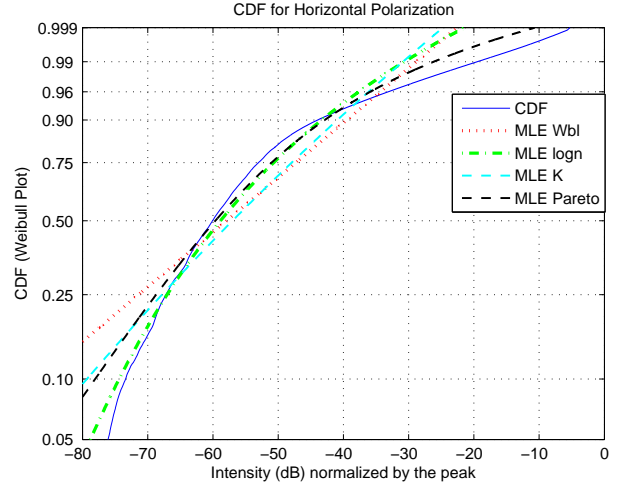| CDF value	0.25	0.5	0.75	0.90	0.99	0.999
Weibull	1.45	0.71	1.78	1.38	3.15	7.57
Lognormal	0.17	0.07	0.54	0.37	2.57	5.16
K	0.45	0.77	0.86	1.4	3.60	7.83
WW	0.17	0.23	0.06	0.27	0.14	3.10
KK	0.17	0.12	0.11	0.20	0.21	2.50
P	0.59	0.05	0.52	0.37	1.73	2.53

Fig. 4. CDF plots of the actual data and estimated Weibull, K and lognormal distributions HH
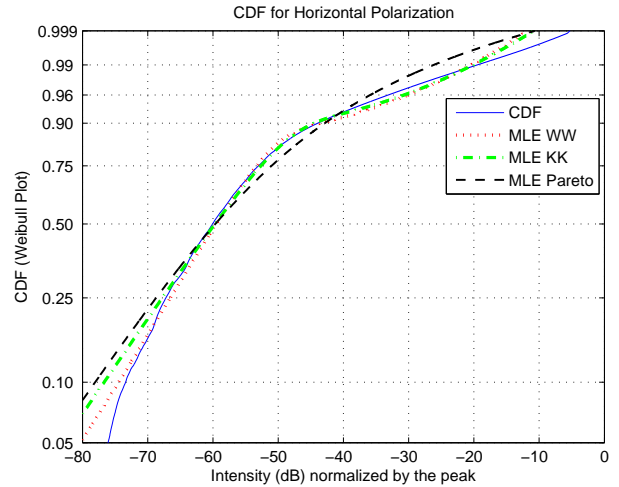

Fig. 5. CDF plots of the actual data and estimated WW, KK and pareto distributions for HH
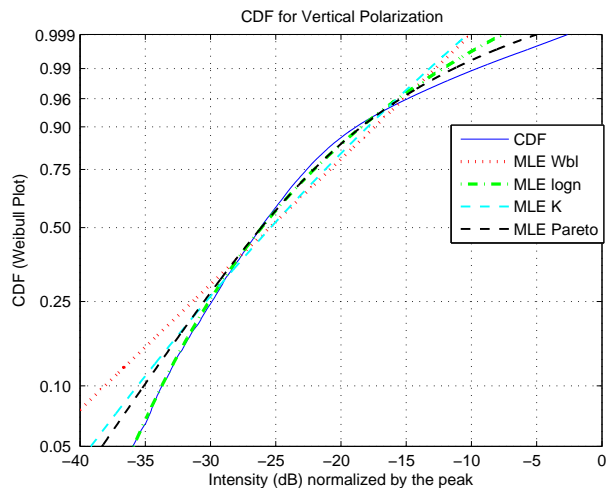
Fig. 6. CDF plots of the actual data and estimated Weibull, K and lognormal distributions VV
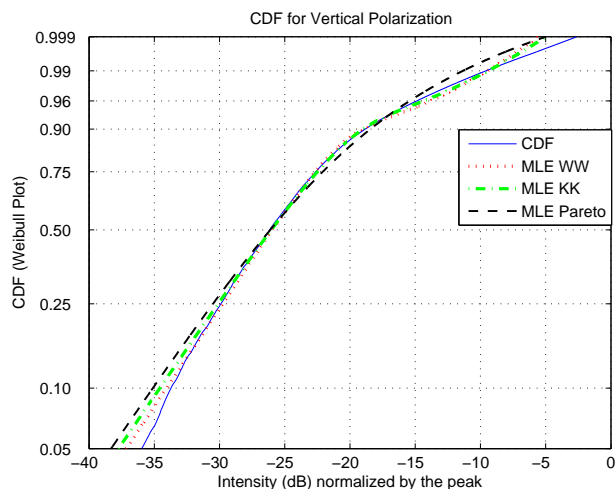

Fig. 7. CDF plots of the actual data and estimated WW, KK and pareto distributions for VV

is analytically tractable and simpler than the K distribution as well as the compound-Gaussian distributions in [13][14]. For the analysis of some radar systems, this could possibly lead to closed formed solutions further down in the detection chain. The Pareto distribution has also been noted in computer networks and other fields for a phenomenon that is analogous to sea clutter. While a general trend has been pushing sea clutter into multiple parameter distributions (five in the case of WW, KK and KA), the two parameter Pareto distribution had comparable performance to the WW and KK distribution for our data set.

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